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Experimental Study of Disp Modulational Instability of Su Waves on Constant Vorticity

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Experiments for negatively sheared current

b)



Linear background current: $U = U_0 + \Omega z$.

Tilde denotes surface current reference frame: $\omega = \tilde{\omega} +$

Waves have potential: $\boldsymbol{u} = U(z)\boldsymbol{\hat{i}} + \boldsymbol{\nabla}\phi$,

Governing equations and boundary condition

Laplace:
$$\nabla^2 \phi = 0$$
 $-d < z < \eta(x, t)$

Kinematic free surface boundary condition: $\eta_t + (\Phi_x + \Omega \eta)\eta_x - \Phi_x$ Dynamic free surface boundary condition: $\Phi_t + \frac{1}{2}\Phi_x^2 + \frac{1}{2}\Phi_z^2 + \Omega \eta_z$

Free surface values: $\Psi \equiv \psi(z = \eta(x, t))$ $\Phi \equiv \phi(z = \psi(z))$

Laboratory experiments (UCL)





Velocity profiles



Linear dispersion relationship: $\tilde{\omega}_0^2 + (\tilde{\omega}_0 \Omega - gk)$



Vor-NLSE

Scaled space and time: $\xi = \epsilon (\tilde{x} - \tilde{c}_g t)$ $\tau = \epsilon^2 t_1$ NLSE: $iA_\tau + LA_{\xi\xi} - M|A|^2 A = 0.$ Coefficients: $L = -\frac{\tilde{\omega}_0(1+\bar{\Omega})^2}{k_0^2(2+\bar{\Omega})^3}$ and $M = \frac{\tilde{\omega}_0 k_0^2}{8(1+\bar{\Omega})}$ $\bar{\Omega} = \Omega/\tilde{\omega}_0$

From envelope to free surface: $\eta^{(1)} = \operatorname{Re} \left| \epsilon A(\xi, \tau) e^{i(k_0 \tilde{x} - \sigma)} \right|^{-1}$

Linear stability analysis



$$A = [a_0 + \delta(\tau, \xi)]e^{-iMa_0^2\tau} \qquad \tilde{\gamma} = \pm \sqrt{K^2 L(K)}$$

Matrix of experiments

$\Omega (\mathrm{s}^{-1})$	$\omega (\mathrm{rad}\mathrm{s}^{-1})$	ka_0
0	7.62	0.15
-0.21	7.17	0.12
-0.48	6.63	0.10



Example time series

Example time spectra

Combined upper and lower sideband: $\Omega = 0$

Combined upper and lower sideband: Ω –() $\hat{K} = 0.59$ $\hat{K}=0.74$ $\hat{K} = 0.88$ $\hat{K} = 1.03$ (a) (b) (c) (d) 0.180.18 0.18 0.18 $\hat{Y}_{\delta}/\hat{Y}_{0}^{0.14}$ **±**0.14 **±**0.14).140.1 0.1 0.1 0.1<u>-</u> 0.06 0.06 0.06 0.06 8 8 44 $\mathbf{4}$ 40 0 0 0 $\hat{K} = 1.33$ $\hat{K} = 1.77$ $\hat{K} = 1.48$ $\hat{K} = 1.62$ (g) (f) (h) (i) 0.18 0.180.18 0.18 $\hat{Y}_{\delta}/\hat{Y}_{0}^{0}$ 0.140.140.140.10. 0.1° 0.10.06L 0.06 - 0.06 0.06 8 8 8 $\mathbf{4}$ 0 4 4 4 0 0 $\hat{K} = 2.07$ (k) (l) $\hat{K} = 2.21$ (m) $\hat{K} = 2.36$ x/λ_0 0.18 0.180.18 $\hat{Y}^0_{\ell} \hat{Y}^0_{\ell} \hat{Y}^0_{\ell}$ 0.140.140.10.10.1±0.06∟ 0 0.06 0 0.06 8 8 4 4 4 8 0 x/λ_0 x/λ_0 x/λ_0

Maximum amplification

FIGURE 11. Maximum amplification factors, denoting the ratio be amplitudes at the first and final gauges, as a function of the nor parameter $\hat{K} = K/\left(a_0\sqrt{-M^*/L^*}\right)$ and for the three shear rates

Conclusions

- Can robustly observed shear-modified linear dispersion relationship currents).
- Negative shear stabilizes the modulational instability: vor-NLSE be

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