Faculty of Science and Engineering Theorem School of Engineering, Computing and Mathematics

2020-05-04

# Experimental study of dispersion and modulational instability of surface gravity waves on constant vorticity currents

van den Bremer, T

https://pearl.plymouth.ac.uk/handle/10026.1/22354

10.5194/egusphere-egu2020-2322 Copernicus Publications

All content in PEARL is protected by copyright law. Author manuscripts are made available in accordance with publisher policies. Please cite only the published version using the details provided on the item record or document. In the absence of an open licence (e.g. Creative Commons), permissions for further reuse of content should be sought from the publisher or author.

# **Experimental Study of Dist** Modulational Instability of St **Waves on Constant Vorticit**

#### James N. Steer<sup>1,5</sup><sup>†</sup>, Alistair G. L. Borthwick<sup>1</sup> Eugeny Buldakov<sup>3</sup> and Ton S. van d

<sup>1</sup>School of Engineering, University of Edinburgh, Edin <sup>2</sup>School of Water, Energy and Environment, Cranfield Univers <sup>3</sup>Department of Civil, Environmental and Geomatic Engineerin Chadwick Building, London, WC1 6 <sup>4</sup>Department of Engineering Science, University of Oxford <sup>5</sup>Wind and Marine Energy Systems Center for D

## Experiments for negatively sheared current

 $b)$ 





Tilde denotes surface current reference frame:  $\omega = \tilde{\omega} +$ 

Waves have potential:  $\mathbf{u} = U(z)\hat{\mathbf{i}} + \nabla \phi$ ,

#### Governing equations and boundary condition

Laplace: 
$$
\nabla^2 \phi = 0 \quad -d < z < \eta(x, t)
$$

Kinematic free surface boundary condition:  $\eta_t + (\Phi_x + \Omega \eta) \eta_x - \Phi_y$  $\Phi_t + \frac{1}{2}\Phi_x^2 + \frac{1}{2}\Phi_z^2 + \Omega r$ Dynamic free surface boundary condition:

Free surface values:  $\Psi \equiv \psi(z = \eta(x, t))$   $\Phi \equiv \phi(z = t)$ 

# Laboratory experiments (UCL)





## Velocity profiles



# Linear dispersion relationship:  $\tilde{\omega}_0^2 + (\tilde{\omega}_0 \Omega - g k)$



#### Vor-NLSE

Scaled space and time:  $\xi = \epsilon(\tilde{x} - \tilde{c}_g t)$   $\tau = \epsilon^2 t$ . NLSE:  $iA_{\tau} + LA_{\xi\xi} - M|A|^2 A = 0.$ Coefficients:  $L = -\frac{\tilde{\omega}_0 (1 + \bar{\Omega})^2}{k_0^2 (2 + \bar{\Omega})^3}$  and  $M = \frac{\tilde{\omega}_0 k_0^2}{8(1 + \bar{\Omega})^3}$  $\bar{\Omega} = \Omega/\tilde{\omega}_0$ 

From envelope to free surface:  $\eta^{(1)} = \text{Re} \left[ \epsilon A(\xi, \tau) e^{i(k_0 \tilde{x} - \xi)} \right]$ 

### Linear stability analysis



$$
A = [a_0 + \delta(\tau, \xi)]e^{-iMa_0^2\tau} \qquad \tilde{\gamma} = \pm \sqrt{K^2L(K)}
$$

## Matrix of experiments





#### Example time series



# Example time spectra



#### Combined upper and lower sideband:  $\Omega = 0$







#### Combined upper and lower sideband:  $\Omega$ - C  $\hat{K}=0.59$  $\hat{K}=0.74$  $\hat{K}=0.88$  $\hat{K}=1.03$  $(a)$  $(b)$  $(c)$  $(d)$  $0.18$  $0.18$  $0.18$  $0.18$  $\frac{1}{2}$  0.14<br>  $\frac{1}{2}$  0.1  $\mathbf{I}0.14$  $\pm 0.14$  $0.14$  $0.1$  $0.1$  $0.1$ <sup> $-$ </sup>  $0.1$  $0.06$  $-0.06$  $0.06$  $0.06$  $\overline{8}$  $\overline{8}$  $\overline{8}$  $\overline{4}$  $\overline{4}$  $\overline{4}$  $\overline{4}$  $\theta$ 0  $\overline{0}$  $\boldsymbol{0}$  $\hat{K}=1.33$  $\hat{K} = 1.77$  $\hat{K} = 1.48$  $\hat{K}=1.62$  $(f)$  $(g)$  $(h)$  $(i)$  $0.18$  $0.18$  $0.18$  $0.18$  $\frac{1}{2}$  0.14<br>  $\frac{1}{2}$  0.1 0.14 0.14 0.14  $0.1$  $\overline{0}$ .  $0.1$  $0.1$  $0.06\Box$  $0.06$  $0.06$  $-0.06$  $\overline{8}$  $\overline{8}$  $\overline{8}$  $\overline{4}$  $\sigma$  $\overline{4}$  $\overline{4}$  $\overline{4}$  $\boldsymbol{0}$  $\boldsymbol{0}$  $\hat{K}=2.07$  $(k)$  $(1)$  $\hat{K} = 2.21$  $(m)$  $\hat{K}=2.36$  $x/\lambda_0$  $0.18$  $0.18$  $0.18$  $\frac{1}{2}$  0.14<br>  $\frac{1}{2}$  0.14  $0.14$  $0.14$  $0.1$  $0.1$  $0.1$  $\frac{1}{2}0.06$  $-0.06$  $0.06$  $\overline{8}$  $\overline{8}$  $\overline{4}$  $\overline{4}$  $\overline{4}$ 8  $\Omega$  $x/\lambda_0$  $x/\lambda_0$  $x/\lambda_0$

#### Maximum amplification



FIGURE 11. Maximum amplification factors, denoting the ratio b amplitudes at the first and final gauges, as a function of the nor parameter  $\hat{K} = K / (a_0 \sqrt{-M^* / L^*})$  and for the three shear rates

# **Conclusions**

- Can robustly observed shear-modified linear dispersion relationship currents).
- Negative shear stabilizes the modulational instability: vor-NLSE better than  $N$

Steer, J.N, A.G.L. Borthwick, D. Stagonas, E. Buldakov and T.S. van study of dispersion and modulational instability of surface gravity wave Journal of Fluid Mechanics, 884, A40.